

Copy 20 of 50

AD615882

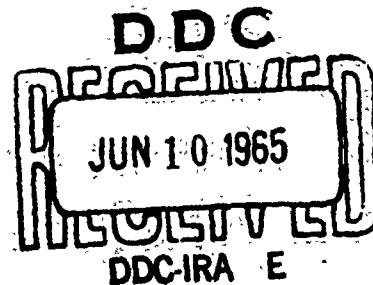
University of California, San Diego  
Marine Physical Laboratory of the  
Scripps Institution of Oceanography  
San Diego 52, California

Internal Memorandum

THE MOTION OF A SPAR BUOY IN SWELL

PART I

Philip Rudnick



Sponsored by  
Office of Naval Research  
Contract Nonr 2216(05)

14 June 1960

COPY	/	OF	/	18
HARD COPY				\$ 1.00
MICROFICHE				\$ 0.50
				40

Reproduction in whole or in part is  
permitted for any purpose of the  
United States Government

MPL-U-17/60

PROCESSING COPY  
ARCHIVE COPY

## THE MOTION OF A SPAR BUOY IN SWELL

## PART I

Philip Rudnick

Suppose the sea surface to be propagating unidirectional, single-frequency gravity waves, so that its upward displacement is given by

$$a_0 \cos(kx - \omega t) \quad \text{with } gk = \omega^2 \quad (1)$$

This perturbed surface is isobaric; the lower isobaric surfaces are similarly perturbed, in the same phase, but with amplitudes showing the usual attenuation with depth. Hence the wave perturbation of pressure at any depth  $d$  below the unperturbed sea surface is

$$p = \rho g a_0 e^{-kd} \cos(kx - \omega t) \quad (2)$$

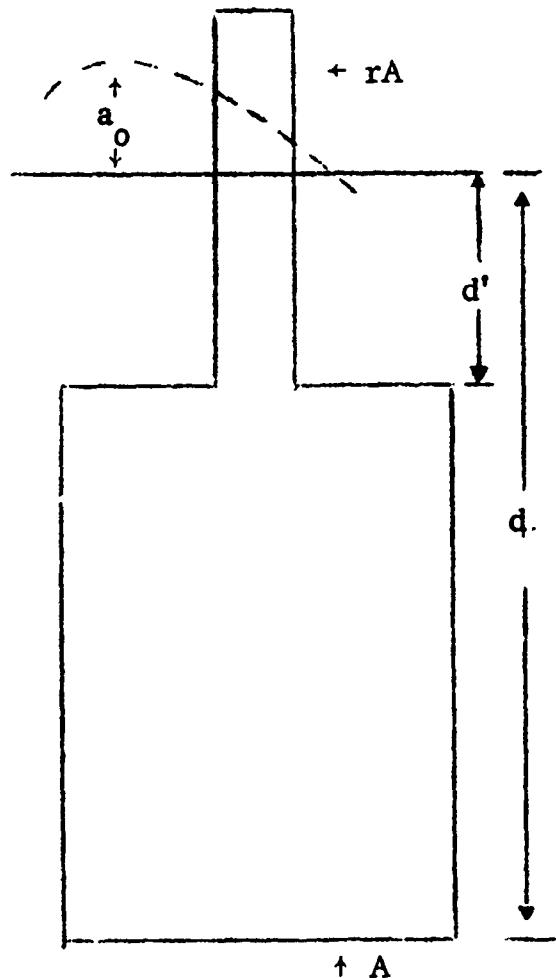
The body forces on a buoy due to these pressure perturbations will now be considered.

It can be said at the outset and in general that these forces are just such as to cause the displaced water to participate fully in the wave motion. Hence a buoy whose mass and mass distribution are closely similar to that of the displaced water will tend to participate fully in the water wave motion, subject of course to the limitation that the latter is not a rigid-body motion. To achieve reduction of buoy motion relative to wave motion, several possible means may be considered.

- a. The mass distribution of the buoy may be made substantially different from that of the displaced water, subject to equality of the two masses.
- b. The configuration of the buoy may be so chosen that its rigidity is a useful constraint.
- c. The fact that the buoy extends through the surface has useful implications.

## Vertical Motion

A relatively simple, but useful, problem is obtained by assuming the buoy to maintain a vertical attitude, and to investigate its vertical motion under pressure forces alone, neglecting damping. Assume the form to be a totally submerged vertical cylinder of cross-section  $A$  surmounted by another of smaller section  $rA$  which extends through the surface. Let the mass of the buoy be  $\rho Ah$ , and let it float in still water with the bottoms of the small and large cylinders at respective depths  $d'$  and  $d$ . In the presence of the surface waves described by Equation (1), the total pressure force acting on the horizontal surfaces is, approximately, using Equation (2)



$$\begin{aligned} F &= A\rho g(d - y + a) - A(1 - r)\rho g(d' - y + a') \\ &= -Ar\rho gy + A\rho g[a - (1 - r)a'] \end{aligned} \quad (3)$$

where  $y$  is a small displacement of the buoy from the still-water equilibrium position, and

$$\begin{aligned} a &= a_0 e^{-kd} \cos(kx - \omega t) \\ a' &= a_0 e^{-kd'} \cos(kx - \omega t) \end{aligned}$$

The second term in the right member of (3) will be treated as the

impressed force and the first as a restoring force, which depends on the displacement of the buoy relative to the unperturbed surface. The equation of motion is

$$m\ddot{y} = F$$

or

$$h\ddot{y} + rgy = g[a - (1 - r)a'] \quad (4)$$

The steady state response at radian frequency  $\omega$  follows from

$$-h\omega^2 y + rgy = -hgky + rgy = g[a - (1 - r)a']$$

or

$$(y/a_0) = \{[1 - r - e^{-k(d-d')}] / [kh - r]\} e^{-kd'} \cos(kx - \omega t) \quad (5)$$

Equation (5) shows that there are two important considerations in choosing  $r$  to produce small  $y$ . One is to avoid the resonant condition  $r = kh$ ; the other is to minimize the larger wave pressure represented by  $a'$ . Taking  $h = 100$  m and  $k = 0.04$  m<sup>-1</sup> (10 sec wave period),  $kh = 4$  and  $r = 1$  gives  $(y/a_0) = 0.02$ . The attenuation is even greater for shorter periods.  $r = 1$  fails to be a good solution as  $kh \rightarrow 1$  (for 20 sec wave period). However, even here some possibility of attenuation exists, for example one gets  $y = 0$  by setting  $r = 0.55$  when  $k(d - d') = 0.8$ .

#### Horizontal Motion

There do not appear to be comparable possibilities for offsetting the horizontal components of the wave pressure forces. About the only obvious means is to have a sufficient length of the buoy extending below the zone of wave action so that its mass and drag will reduce its motion below that of the near-surface water. This will evidently be effective for periods below 10 sec but not at 20 sec.

PR:sg  
Stencil  
1-17-63